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Mathematical Culture in the Landscapes of West Lake

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Abstract

West Lake is renowned worldwide for its picturesque scenery and profound cultural heritage. This paper uses two famous scenic spots, Leifeng Pagoda and Broken Bridge, as examples to explore the mathematical culture embedded in the landscapes of West Lake. We analyze not only the architectural structure and geometric aesthetics of Leifeng Pagoda and Broken Bridge but also employ mathematical knowledge to understand these structures. This includes using the theorem of similar triangles to measure the height of Leifeng Pagoda, examining the geometric properties of Broken Bridge, such as parallelism, perpendicularity, and symmetry, and solving the coordinate system and parabolic equations related to Broken Bridge. Through these case studies, we discover that West Lake is not only a treasure of natural landscapes but also a fusion of humanistic spirit and scientific wisdom. This paper aims to enrich the understanding of traditional Chinese culture through the exploration of the mathematical culture in West Lake's landscapes and to provide new perspectives and ideas for the future preservation of cultural heritage and scientific education.

Subject Areas

Mathematical Models, Applied Mathematics, Plane Geometry

Keywords

Similar Triangles, Parabola, West Lake Landscapes, Leifeng Pagoda, Broken Bridge

1. Introduction

West Lake, located in the Xihu District of Hangzhou, Zhejiang Province, China, is renowned worldwide for its beautiful landscapes, earning the nickname "Heaven on Earth". According to official records, the West Lake Scenic Area is divided into five regions: Lakeside, Central Lake, Northern Mountains, Southern Mountains, and Qiantang. The total area of the scenic area is 59.04 square kilometers, with a catchment area of 21.22 square kilometers and a lake surface area of 6.38 square kilometers. The West Lake Scenic Area can be categorized into natural and artificial landscapes based on their formation. Common artificial landscapes include islands and causeways such as Xiao Yingzhou, Su Causeway, Bai Causeway, and Yanggong Causeway, upon which people have developed structures like pagodas, bridges, and pavilions.

Through our field visits to the West Lake Scenic Area, we discovered that the beauty of West Lake's landscapes is all-encompassing, derived from nature and refined through craftsmanship. It not only boasts unique natural scenery but also has a rich cultural history. In this perfect blend of natural and cultural landscapes, mathematical culture also plays an important role. This paper will explore the applications of mathematics in West Lake, including using the theorem of similar triangles to measure Leifeng Pagoda, examining the geometric properties of Broken Bridge, and solving the parabolic equation related to Broken Bridge.

The purpose of this research is to demonstrate how mathematical principles can be applied to understand and appreciate the intricate details of West Lake's landscapes. By delving into these mathematical aspects, we aim to highlight the intrinsic connections between mathematics and the aesthetic and cultural values of West Lake, thereby providing a new perspective on the appreciation of this UNESCO World Heritage site.

2. Leifeng Pagoda and Mathematical Culture

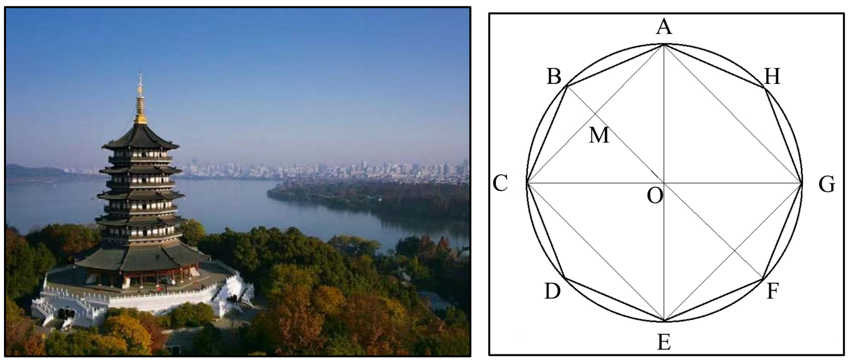
2.1. Introduction to Leifeng Pagoda

Leifeng Pagoda is one of the important landmarks of West Lake, situated on the south bank atop Sunset Hill. Historical records state that it was built by King Qian Chu of the Wuyue Kingdom to enshrine the Buddha's relics and pray for national peace and prosperity. The pagoda was originally constructed in the second year of the Taiping Xingguo period of the Northern Song Dynasty (977 AD) and has been reconstructed several times throughout history. The current structure, rebuilt in 2002, was designed based on the original Leifeng Pagoda and is one of China's nine famous pagodas. It is the first colored copper sculpture pagoda in China, with nearly 20,000 copper tiles, covering an area of 2370 square meters. It is the largest copper pagoda in terms of the use of copper components and the area of copper decoration. The main structure of Leifeng Pagoda follows the style of Tang and Song dynasty pavilions, with a total height of 71.679 meters and a total area of 3133 square meters. The diameter of the pagoda body is 28 meters, the side length is 11 meters, and the perimeter is 88 meters. The base of the pagoda is the original site of Leifeng Pagoda [1].

2.2. Leifeng Pagoda and the Regular Octagon

The floor plan of Leifeng Pagoda is a regular octagon (See **Figure 1(a)**), with each layer consisting of eight sides. The characteristics of a regular octagon include each internal angle being 135 degrees, equal side lengths, and a total internal angle sum of 1080 degrees. Such octagonal structures are not uncommon in ancient Chinese architecture. For instance, the Sakyamuni Pagoda of Fogong Temple in Yingxian, Shanxi, also features an octagonal structure like Leifeng Pagoda. Octagonal designs are also present in Western architecture, such as the Baptistery of San Giovanni in Florence from the Italian Renaissance period and the pedestal of the Statue of Liberty in modern New York, USA.

On the right side of Figure 1(b), we show a regular octagon and its circumscribed circle. It can be observed that any line segment connecting two points through the center of the circle serves as an axis of symmetry for the regular octagon. This perfect symmetry gives Leifeng Pagoda a harmonious and unified appearance from any angle. Moreover, the regular octagon exhibits its aesthetic through both rotational symmetry and axial symmetry. This symmetry not only enhances the visual appeal of Leifeng Pagoda but also provides balanced force distribution in its structure.



(a) (b)

Figure 1. (a) Real view of Leifeng Pagoda, (b) Octagonal structure of Leifeng Pagoda.

2.3. Leifeng Pagoda and the Regular Octagon

According to official records, Leifeng Pagoda has eight levels, including two levels below the base (one of which is underground), and the pagoda body above the base has five levels, with a total height of 71.679 meters. The base of Leifeng Pagoda has a diameter (across opposite points) of 60 meters, with a side length of 23.34 meters and a perimeter of 186.72 meters. The auxiliary base has a diameter of 35.25 meters, with a side length of 13.426 meters and a perimeter of 107.408 meters. The pagoda body has a diameter of 28 meters, with a side length of 11 meters and a perimeter of 88 meters.

The diameter (across opposite points) of the base or the body of the pagoda refers to the straight-line distance passing through the center of the pagoda and connecting two opposite points. Since the body of Leifeng Pagoda is in the shape of a regular octagon, the diameter refers to the diameter of the circumscribed circle of the regular octagon. We can calculate the side length and perimeter from the diameter data. The calculation process is as follows:

Given that the diameter (D) of the base (circumscribed circle diameter) is 60 meters, we can calculate the radius (r) of the circumscribed circle using the formula:

$r = D/2 = 60/2 = 30$ meters

Next, we calculate the side length(s) of the regular octagon using the formula for the side length of a regular octagon inscribed in a circle:

$r = a/(2\sin(\pi/8))$,

$a = 2r\sin(\pi/8) \approx 2 \times 30 \times 0.3827 = 22.962$ meters

$P = 8a = 8 \times 22.962 \approx 183.696$ meters

This confirms the given perimeter for the base. Similarly, for the auxiliary base and the pagoda body, we can apply the same formulas using their respective diameters to calculate the side lengths and perimeters, verifying the provided values.

2.4. Leifeng Pagoda and Similar Triangles

The principle of similar triangles can be used to measure the height of a building [2]. Here, we used three different methods to construct similar triangles and subsequently measured the approximate height of Leifeng Pagoda.

1) Using a Flagpole to Construct Similar Triangles

As shown in **Figure 2**, we erected a flagpole at a certain point on the ground (denoted as point C) with a height of 2 meters (represented as CD in the figure). We ensured that the top of Leifeng Pagoda (point B), the top of the flagpole (point D), and a point on the ground (point E) were aligned in a straight line. The distance CE was easily measured to be 3 meters. Next, we moved the flagpole to another direction, maintaining the alignment of the top of Leifeng Pagoda (point B), the top of the flagpole (point H), and a point on the ground (point F) in a straight line. The height of the flagpole GH remained constant, and we measured GF to be 5 meters, with the flagpole moved 60 meters (CG) [3]. Based on the principle of similar triangles, we can calculate the approximate height of Leifeng Pagoda. The specific calculation process is as follows:

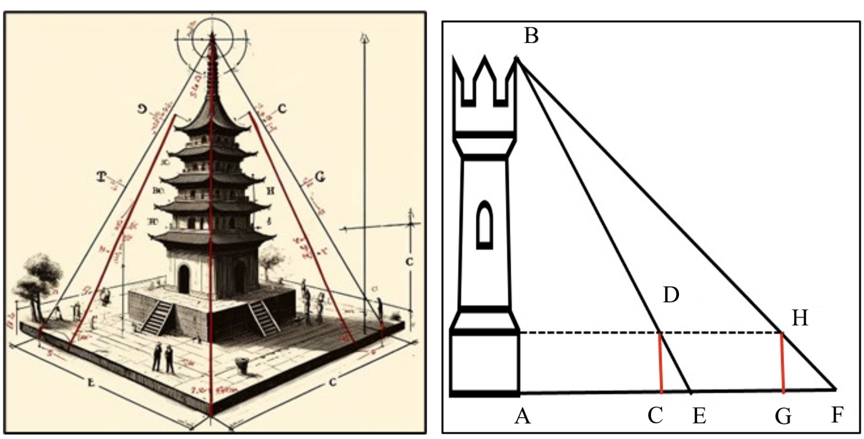


Figure 2. Measurement of Leifeng Pagoda and Flagpole.

∵CD // AB, ∴∠EDC = ∠EBA, ∠BEA = ∠DEC, ∴ΔABE~ΔCDE

$$\therefore CD/AB = CE/AE, \text{ so } 2/AB = 3/(3 + AC) \quad (1)$$

for the same reason, GH // AB, ∴ΔABF~ΔGHF

$$\therefore GH/AB = GF/AF, 2/AB = 5/(5 + 60 + AC) \quad (2)$$

Combining Equations (1) and (2), we can calculate that AC is 90 meters and the height of Leifeng Tower AB is 62 meters.

2) Using Shadows to Construct Similar Triangles

On a sunny day, we observed the shadow of Leifeng Pagoda and placed a flagpole with a height of 2 meters at the top of its shadow. Utilizing the principle that sunlight rays are parallel, we can construct two similar triangles. The specific calculation process is as follows:

As shown in Figure 3, we have:

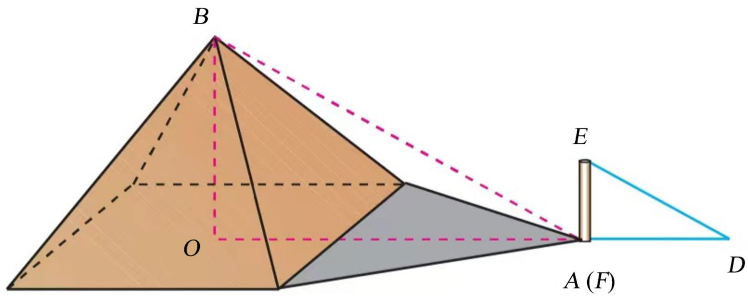


Figure 3. Projection with Leifeng Pagoda.

∵OB // AE, ∠BOA = ∠EAD = 90^o, ∠BAO = ∠EDA, ∴ΔOAB~ΔADE

We measure the distance AD to be 3 meters and the distance OA to be about 100 meters.

According to the properties of similar triangles, we have OB/AE = (OA + AD)/AD

That is, OB/2 = 103/3, and the height of Leifeng Tower OB is approximately 68.7 meters.

3) Using a Mirror to Construct Similar Triangles

Firstly, on a sunny morning, we placed a flat mirror at a certain distance from the pagoda, noted as point O. The top of Leifeng Pagoda is marked as point A, and the base is marked as point B. We positioned a screen with a height of 2 meters behind the mirror, with the top of the screen noted as point D and the bottom as point C. By adjusting the distance, we ensured that sunlight reflected off the mirror reached the top point D. According to the law of reflection, the angle of incidence equals the angle of reflection, so we obtain the similar triangles ΔOAB~ΔOCD. (See Figure 4)

Given as follows:

1) The distance from the mirror to the base of the pagoda (OB) = 32 meters

2) The distance from the mirror to the base of the screen (OC) = 1 meter

3) The height of the screen (CD) = 2 meters

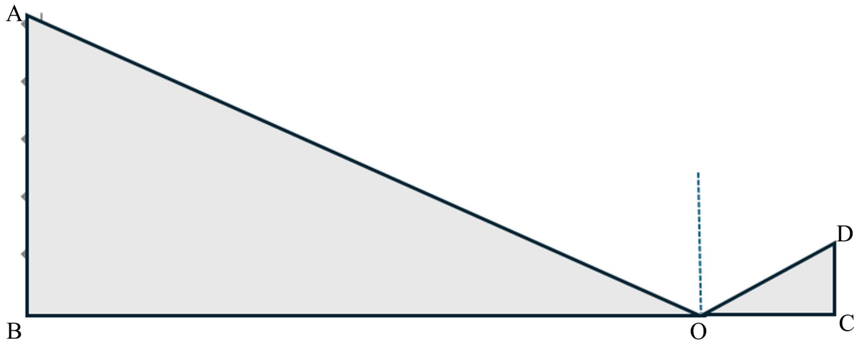


Figure 4. Mirror and Leifeng Pagoda.

Since the triangles are similar, the ratio of their corresponding sides is equal:

OB/OC = AB/CD

Therefore: 32/1 = AB/2

Solving for AB (the height of Leifeng Pagoda):

$AB = 32 \times 2 = 64$ meters

Thus, using the mirror method, we calculate the height of Leifeng Pagoda to be approximately 64 meters.

2.5. Error Analysis

Using three different methods to construct similar triangles, we estimated the approximate height of Leifeng Pagoda to be 62 meters, 68.7 meters, and 64 meters, respectively. According to official records, the actual height of Leifeng Pagoda is 71.679 meters. All three estimation methods exhibit some degree of error, and it is necessary to analyze the reasons for these errors.

Method 1: Using a Flagpole to Construct Similar Triangles

In the ideal scenario, the top of Leifeng Pagoda, the top of the flagpole, and a point on the ground should align in a straight line. Without the aid of professional instruments like a laser level, it is challenging to ensure these three points are perfectly collinear. This misalignment is a primary source of error in the first method's estimation.

Method 2: Using Shadows to Construct Similar Triangles

When using sunlight to construct similar triangles, factors like the refractive index of the air can affect accuracy. Additionally, measuring the distance from the internal center point of Leifeng Pagoda to the top of its shadow can introduce significant errors. These factors contribute to the discrepancy between the estimated and actual heights.

Method 3: Using a Mirror to Construct Similar Triangles

First, observing the refraction of light with the naked eye can lead to inaccuracies. Second, it is difficult to precisely adjust the top of the screen to coincide with the light reflected from the mirror. Furthermore, like Method 2, measuring the distance from the internal center point of Leifeng Pagoda to an external point can also introduce errors.

In summary, the main sources of error across all three methods include alignment issues, atmospheric conditions affecting light paths, and difficulties in precise distance measurement. These factors collectively contribute to the differences between our estimated values and the actual height of Leifeng Pagoda.

3. Broken Bridge and Mathematical Culture

In the West Lake Scenic Area of Hangzhou, there are approximately 17 bridges connecting various parts of the scenic area, forming beautiful landscape links. Among these bridges, the most famous is the Broken Bridge, which is the only bridge among them listed as one of the Ten Scenes of West Lake, known as the "First Bridge." The Broken Bridge is located at the dividing point between the North Inner Lake and the Outer West Lake. One end of the bridge spans Beishan Road, while the other connects to Bai Causeway. According to legend, the Broken Bridge was originally built during the Tang Dynasty, referred to as Baoyou Bridge during the Song Dynasty, and Duanjia Bridge during the Yuan Dynasty. After the Yuan Dynasty, it was also known as "Short Bridge" to contrast with the Long Bridge, leading to the name "Broken" being a homophonic variation of "Duan" (section). We will use the Broken Bridge as an example to explore the mathematical culture embedded within it [4].

3.1. Parallelism, Perpendicularity, and Symmetry in the Broken Bridge

The Broken Bridge is an arched single-span ring hole stone bridge, approximately 8.8 meters long and 8.6 meters wide, with a single-span net crossing of 6.1 meters. It is constructed from blue bricks and white tiles. The mathematical properties of the Broken Bridge can be analyzed as follows:

Parallelism

The top and bottom edges of the bridge deck are parallel, ensuring a consistent width along the bridge's length. This parallelism is essential for the structural stability and aesthetic harmony of the bridge.

Perpendicularity

The pillars and supports of the bridge are perpendicular to the ground, providing the necessary vertical support to bear the load of the bridge and the traffic passing over it. Perpendicularity ensures that the weight is evenly distributed and transferred to the ground.

Symmetry

The Broken Bridge exhibits bilateral symmetry, meaning that one half of the bridge is a mirror image of the other. This symmetry not only enhances the visual appeal of the bridge but also contributes to its structural balance and stability. The arch of the bridge is symmetric around its central axis, creating an aesthetically pleasing and structurally sound design.

These geometric properties are crucial in the design and construction of the Broken Bridge, reflecting the sophisticated mathematical understanding and engineering skills of the builders [5].

From the elevation view of the Broken Bridge, we see that the main beam of the bridge is a horizontal line, with railings and bridge piers forming vertical lines, extending horizontally along the bridge structure. The railings on the bridge surface are parallel to each other and perpendicular to the bridge deck. The bridge piers are perpendicular to the bridge deck, ensuring the structural strength of the bridge. This perpendicular relationship plays a crucial role in the design and construction of the bridge.

Additionally, the design of the Broken Bridge reflects a significant amount of symmetrical aesthetics. Firstly, both sides of the bridge maintain symmetry in structure and decoration, enhancing the visual harmony. Symmetrical figures are formed around the central point of the bridge, such as the symmetry of the railings, the arch, and the piers. The arch of the bridge approximates a semicircle, a design that not only functions for stability and load bearing but also visually presents extreme symmetry. The semicircular arch, like a perfect curve, achieves structural balance and makes the arch visually harmonious [6].

When sunlight falls on the surface of West Lake, the reflection of the arch in the water forms a complete circle, further enhancing the visual appeal of the Broken Bridge [7]. Symmetry is an important principle in aesthetic design. The symmetrical design of the bridge arch aligns with people's intuitive perception of beauty and makes the bridge appear more harmonious within the natural environment. Whether from the geometric shape of the arch or its reflection in the lake, the unique charm of symmetrical beauty is evident [8]. This symmetry is not just a visual pleasure but also conveys a sense of stability and balance through geometric shapes. The semicircular design of the Broken Bridge arch is undoubtedly a classic example of the perfect integration of mathematical beauty and natural landscape.

3.2. The Radius and Circle of the Broken Bridge

The radius of the bridge arch, along with the length of the bridge deck and the height of the bridge, maintains a proportional relationship that makes the overall design of the Broken Bridge appear balanced and harmonious, thus enhancing its aesthetic value. Based on field surveys, the height of the bridge arch is approximately 4.3 meters, and the width of the bridge deck is 8.6 meters. We can briefly analyze the geometric characteristics of the bridge arch.

First, assuming the radius of the bridge arch is equal to its height, $r = 4.3$ meters, we can use the formula for the circumference of a circle to calculate the circumference of the circle formed by the bridge arch and its reflection on the lake surface:

$C = 2\pi r = 2\pi \times 4.3 \approx 27$ meters.

We can also use the formula for the area of a circle to calculate the area:

$S = \pi r^2 = \pi \times 4.3^2 \approx 58$ square meters.

When considering the bridge arch and its reflection forming a complete cylinder on the lake surface, the geometric properties of this structure in three-dimensional space become even more striking. We can use the formula for the volume of a cylinder to calculate it:

$V = \pi \times r^2 \times h = \pi \times 4.3^2 \times 8.6 \approx 499$ cubic meters.

The height of the bridge arch, being 4.3 meters, achieves an ideal balance between functionality and aesthetics for the Broken Bridge. Firstly, this height ensures smooth water flow beneath the bridge, effectively preventing flooding and water accumulation, thereby enhancing the durability and safety of the bridge. At the same time, the height of 4.3 meters provides sufficient clearance for pedestrians and small boats, facilitating transportation and sightseeing activities.

As one of the Ten Scenes of West Lake, the Broken Bridge, especially its "Broken Bridge Snow" view, is famous far and wide. The unique design of the bridge arch undoubtedly adds infinite charm to this landscape.

3.3. Stability of Triangles

The triangle is one of the most stable shapes in geometry because it does not easily deform when external forces are applied. This stability makes triangles widely used in architecture and engineering structures, such as bridges, towers, and roofs. In bridge structures, triangular designs are commonly used in bridge piers, bridge towers, and truss structures. In the three-dimensional sectional view of a bridge, the main part of the bridge forms an isosceles triangle (shown in red in the diagram), and the two bridge piers form right triangles (shown in green in the diagram). Additionally, if the hypotenuses of the two green right triangles are extended upwards, they also form an isosceles triangle (not shown in the diagram). The triangular structures in bridges can effectively distribute and withstand external forces, ensuring the stability and safety of the bridge (See Figure 5).

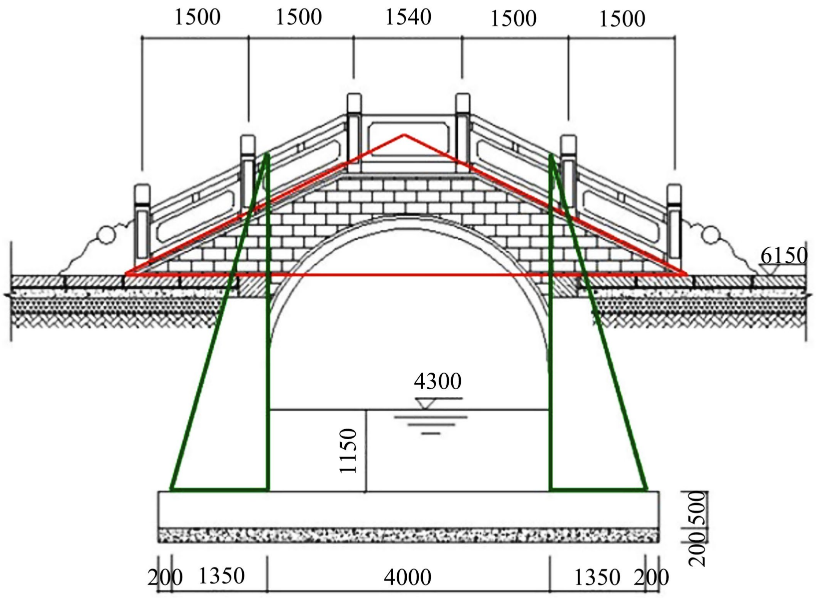


Figure 5. Three-dimensional sectional view of a broken bridge and triangles.

The triangular structures in bridges can effectively distribute and withstand external forces, ensuring the stability and safety of the bridge.

3.4. Broken Bridge and the Coordinate System

We want to draw the coordinate system of the Broken Bridge based on the location of the West Lake. First, we need to determine the location of the Broken Bridge in the West Lake, and then draw the coordinate system on the map based on the relevant information. Broken Bridge is located at the northeast corner of West Lake in Hangzhou, Zhejiang Province, China, and its geographic coordinates are roughly 30.2547 degrees north latitude and 120.1607 degrees east longitude. We use the map tool (Google Maps) to export the boundary data of the West Lake, and then mark the exact location of the Broken Bridge on the drawn map of the West Lake to pinpoint the location according to its geographic coordinates. Finally, we take the center of West Lake as the origin (0, 0) to establish the right-angle coordinate system.

1) Calculate the difference between latitude and longitude

The latitude of Broken Bridge is 30.2547 degrees, the longitude is 120.1607 degrees, the latitude and longitude of the center of the West Lake are 30.2547 degrees and 120.1607 degrees, respectively, the difference of latitude is 0.0012 degrees, the difference of longitude is 0.0007 degrees.

2) Conversion to unit meters

The distance corresponding to the difference in latitude is: 0.0012 degrees * 111 km/degree = 133.2 meters

The distance corresponding to the difference in longitude is: 0.0007 degrees * 111 km/degree = 77.7 meters

3) Calculate the coordinates of the broken bridge

We set the longitude as the X-axis and the latitude direction as the Y-axis, the coordinates of the Broken Bridge relative to the center of the West Lake are (77.7, 133.2).

Accordingly, we draw the coordinate system as follows (See Figure 6).

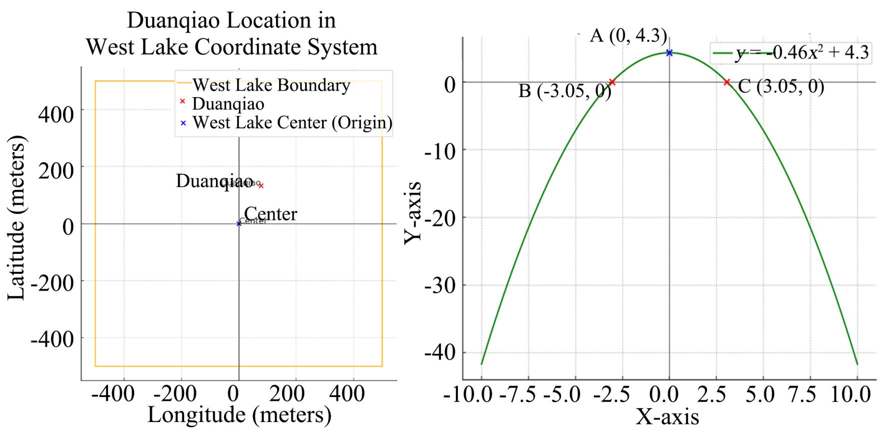


Figure 6. Broken bridge and parabolic equation in coordinate system.

In addition, we calculate the parabolic equation of the broken bridge based on the data of the holes of the broken bridge. According to the data related to the broken bridge, the length is about 8.8 meters, the width is 8.6 meters, and the net span of a single hole is 6.1 meters. We decompose the cross-section of the broken bridge and establish a right-angle coordinate system as shown in **Figure 6** (right). Write the coordinates of a few points on the parabola, with vertex A having the coordinates (0, 4.3), point B having the coordinates (−3.05, 0), and point C having the coordinates (3.05, 0). Assuming that the functional form of the equation of the parabola is $y = ax^2 + bx + c$, substituting the coordinates of the three points into the equation gives $y = -0.46x^2 + 4.3$.

4. Conclusions

The West Lake scenery in Hangzhou is famous all over the world and contains the aesthetic and cultural heritage of generations of people. This paper explores the mathematical culture embedded in the two famous scenic spots of Leifeng Pagoda and Broken Bridge in the landscape of West Lake through the study of these two scenic spots. West Lake is not only a treasure of natural landscape, but also a combination of humanistic spirit and scientific wisdom. When analyzing the architectural structure and geometrical aesthetics of Leifeng Pagoda, we found the symmetry and proportionality embedded in it, which reflect the deep understanding and application of mathematical principles by ancient architects. The study of the Broken Bridge, on the other hand, shows us that ancient literati not only showed their love for natural scenery in literature and art, but also embedded rich mathematical ideas in it, such as the use of coordinate system and distance measurement [9].

Through the case study of Leifeng Pagoda and Broken Bridge, we further realize the important position of mathematical culture in Chinese traditional culture. Mathematics is not only a tool, but also a way to understand the world and explain nature. This mathematical culture is fully reflected between the landscape of West Lake, which makes West Lake not only a scenic spot, but also a carrier of culture and wisdom. Through the excavation and interpretation of its mathematical culture, we can not only appreciate its surface beauty, but also deeply comprehend its deep scientific and cultural connotations. Future research can be further extended to other scenic spots to explore more mathematical cultures hidden behind the scenic spots, providing new perspectives and ideas for our cultural heritage protection and science education.

Conflicts of Interest

The authors declare no conflicts of interest.

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